# Prospect Equality: A Force of Redistribution 

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#### Abstract

Recent evidence demonstrates that the perceived, not the actual, level of income inequality influences the redistribution policy. The perception of inequality, as conceptualized in this paper, is closely related to both objective inequality and prospect equality. An axiomatic system of individual preferences is suggested and demonstrated to characterize an index of perceived inequality. Prospect equality reflects the individual ideal level of equality, and it serves as a reference point for perception. I adopt the proposed notion to study voting on redistribution. I theoretically identify the conditions under which a more equal society will demand redistribution while a less equal society blocks redistribution. These insights help explain the redistribution puzzle observed across nations.


keywords: prospect equality; redistribution; voting.

## 1 Introduction

At least since Meltzer and Richard [1981], many have believed that as income inequality increases, a society will prefer policies supporting greater redistribution to counter excessive income disparities. One early stream of study of inequality focused on the development of an objective measure of inequality as a tool for

[^0]intervening in inequality. ${ }^{1}$ However, empirical support for the claim that objective inequality leads to more redistributive policies is generally ambiguous (Meltzer and Richard [1983], Borge and Rattsø [2004]). In particular, Alesina and Glaeser [2004] observed the opposite pattern: Western European countries have lower levels of objective income inequality than the US but demand a higher redistribution policy. This observation is widely known as the redistribution puzzle. While the emphasis on income inequality as a driver of redistribution appeared to be largely unsupported, more recent studies suggest that a higher perceived level of inequality is positively connected to redistribution policy (Gimpelson and Treisman [2018], Page and Goldstein [2016], Kuhn [2020]). It makes sense that voting behavior would be shaped more by the perception of inequality than by an obscure objective inequality index. Due to cognitive limitations and framing effects, individuals or voters can rarely be guided to correctly track objective levels of income inequality. Therefore, social policy, being an aggregation of individual opinions, must legitimately reflect such limitations. Altogether, this suggests a need to pin down the mechanism of perception formation and, thereby, develop a theory for the measurement of inequality perception. Furthermore, properly incorporating perception with voting on redistribution should help resolve the redistribution puzzle. As a result, this measure would be able to facilitate the separation of societies that are economically similar in terms of income distribution but fundamentally different in terms of redistribution demand. If such an index becomes focal, we may achieve novel insights into redistribution policies.

In this paper, I first address the formation and measurement of individual perceptions of inequality. In fact, we already know of many attributes that play some role in the formation of perception. For instance, social beliefs (Alesina and Angeletos [2005]), experience (Roth and Wohlfart [2018]), and culture (Luttmer and Singhal [2011]) may affect perceived inequality in different contexts. Ideally, a theory must include all attributes that are relevant to the formation of perception. However, I will simplify the analysis enormously by restricting attention to the attributes that potentially have an effective influence on policy making. It is my

[^1]contention that two attributes are closely linked to the generation of perception. The first attribute, unsurprisingly, is objective inequality. As Kerr [2014] demonstrated, in a country, growth in inequality typically produces greater support for redistribution. Therefore, objective inequality has a positive effect on redistribution ceteris paribus. The other attribute, I believe, is prospect equality. Prospect equality is derived from a prospect set, which consists of all the income profiles an individual could expect. In my theory, prospect equality is regarded as the ideal equality level from among the prospect set. As Almås, Cappelen, Lind, Sørensen, and Tungodden [2011], Cappelen, Hole, Sørensen, and Tungodden [2007] demonstrated, the view of what is the ideal or feasible level of equality is also a critical determinant of distribution demand. Putting these two pieces together, I propose that prospect equality serves as a reference point to determine how inequality is perceived. Perceived inequality is higher as the gap between the actual level of inequality and prospect equality widens, all else being equal. Therefore, when an individual has high prospect equality, she may perceive inequality even if the actual level of income equality is high and only slightly lower than her prospect equality. This perception is consistent with the observation that an individual who believes luck determines success may have high prospect equality and demand greater redistribution. When an individual has low prospect equality, then she could perceive inequality even if the actual level of income equality is relatively high. This perception is consistent with the observation that an individual who believes effort determines success may have low prospect equality and, therefore, fight against redistribution. Therefore, my theory can help explain the observation of Alesina and Glaeser [2004] and many other similar observations.

To obtain the exact measurement of perceived inequality in practice, I need to separately measure the inequality between the actual income profile and the prospect profile. Given an income profile $x$, it is straightforward to measure its level of inequality using a classical index $I(x)$, such as the Gini index. The critical part of my theory is how to determine the prospect set and then derive the prospect equality. I argue that prospect, a key determinant of perception, is the ideal equality among all feasible allocations that are politically acceptable from an individual point of view. Prospect sets, however, depend not only on the current political con-
text but also on individual characteristics, such as individual beliefs about social justice. By adding the subjective view that individuals hold of ideal income allocation, my model allows us to incorporate the fact that voters base their voting on redistribution on their believed 'fair' allocation and not just on the actual level of income inequality. Therefore, the prospect set applied here is subjective expectations, as collected by opinion surveys. ${ }^{2}$ Formally, given a pretax income profile, a prospect set $A$ can be imagined as a set of post-tax income profiles that are considered feasible by individuals. Prospect equality is, therefore, regarded as the ideal equality, i.e., $\min _{y \in A} I(y)$. Hence, the perception of inequality is measured by the absolute difference between the actual level of inequality $I(x)$ and (discounted) prospect equality $\min _{y \in A} I(y)$. One may be concerned about my proposal to use subjective data, a prospect set, to justify the preferences. However, I regard it as an advantage rather than disadvantage to explore the perception of inequality. As Manski [2004] pointed out, a proper combination including subjective data would "mitigate the credibility problem and improve the ability to predict behavior".

I further examine how the perception of inequality helps explain the redistribution puzzle by adapting the prospect inequality preferences to analyze voting on redistribution. In particular, I characterize the conditions of redistribution under which an objectively more equal society demands redistribution but an objectively less equal society blocks redistribution, thereby providing a reasonable explanation for the redistribution puzzle.

This paper is closely related to the literature that attempts to understand the determinants of inequality reduction. One stream of study is driven by individual behaviors, and it tends to explain policy making from the angle of individual altruism (Fehr and Schmidt [1999]), prospects of mobility (Benabou and Ok [2001]), belief in fairness (Bénabou and Tirole [2006]) and so on. The other stream of study is driven by political systems such as clientelism (Lizzeri and Persico [2001]) or identity politics (Roemer [1998]). Therefore, my measure of the perception of inequality can, somehow, be regarded as a unified notion of both approaches: the first part of my measure, objective inequality, can be regarded as an inequality reduc-

[^2]tion, and the second part, prospect equality, can be regarded as the scope of political reality. Different from their purely game theoretical analysis, I adopt an axiomatic approach to highlight the normative criteria that shape perception of inequality.

My idea discussed above is well connected to certain developments in decision theory. Specifically, the so-called prospect theory of Kahneman and Tversky [1979] is found to be relevant for the study of perceived inequality. As I claimed above, prospect equality serves as a reference point for evaluating the perception of inequality. Alternatively, the prospect set can be regarded as a тепи studied by Gul and Pesendorfer [2001]. In fact, Dillenberger and Sadowski [2012] is closer to my representation result in that they also incorporate the fairness concern into the prospect set. However, due to the different motivations, it is not immediately clear, in a voting system, how their model can explain the redistribution puzzle. In contrast, my first task in the paper consists of presenting a set of axioms for a measurement and proving a representation result as they did. The analysis used in this endeavor is also related to other work on the axiomatic foundations for the measurement of income profile, which deviates from classic objective measures, notably the conflict measurement of Esteban and Ray [1994]. However, our motivation, application and measurement are far from those of previous studies.

Finally, my paper is closely related to the literature on voting on redistribution. Meltzer and Richard [1981] is the first paper linking inequality and redistribution. In their model, the redistribution policy depends on objective inequality, which is measured as the difference between median income and average income. An alternative method is to add social identity (Akerlof and Kranton [2000]) to individual preferences. Although the social identity approach can potentially explain why a society comprising heterogeneous groups has less demand for redistribution, it lacks theoretical support based on voting on redistribution. More recently, an alternative to adding fairness to individual preferences is the inequality aversion approach (Tyran and Sausgruber [2006], Dhami and al Nowaihi [2010]), which allows preferences to be influenced by the income differences across individuals. They are successful in explaining an observation that an equal society may also support redistribution, but they cannot fully resolve the redistribution puzzle because inequality aversion is not consistent with the observation that some less equal countries only
support slight or even block redistribution.
I organize this paper as follows. Section 2 provides a basic setup and proposes a measure of the perception of inequality. Section 3 develops an axiomatic foundation for the Gini index measure of the perception of inequality. The main result is also obtained. Section 4 studies voting on redistribution. Section 5 concludes the paper, and the Appendix contains all the proofs.

## 2 The Model

An income profile (or allocation) is a list of individuals and a list of corresponding incomes. Specifically, a society consists of $n \geq 2$ individuals, and $x_{i}$ denotes the income of individual $i$ for $i=1, \ldots, n$. An income profile is represented as a finitedimensional vector $x=\left(x_{1}, \ldots, x_{n}\right)$. I denote $X=\mathbb{R}_{+}^{n}$ as the set of all possible income profiles. ${ }^{3}$ For $x \in X$, I write $\mu(x)=\frac{1}{n} \sum_{i=1}^{n} x_{i}$ for the average income of $x$.

Let $\mathscr{A}$ denote the set of nonempty subsets of $X$. Each element $A$ in $\mathscr{A}$ refers to a prospect set of income profiles, representing the desirable income profiles that individuals believe what the income distributions ought to look like. The alternatives in my analysis are the pairs of income profile and the associated prospect set. I analyze how an individual perceives the inequality of income profile $x$ when the prospect set of income profiles is $A$. Formally, let $\mathbb{D}:=X \times \mathscr{A}$ and denote by $\succsim$ the individual preference relation on $\mathbb{D}$. I interpret relation $(x, A) \succsim(y, B)$ such that an individual "perceives less inequality" from profile $x$ with prospect $A$ than from $y$ with prospect $B$. I say that a function $J: \mathbb{D} \rightarrow \mathbb{R}$ represents the individual perception of inequality $\succsim$ if, for all $(x, A),(y, B) \in \mathbb{D}$,

$$
(x, A) \succsim(y, B) \text { if and only if } J(x, A) \leq J(y, B)
$$

In fact, I can restrict the prospect sets further to adapt to different situations. To consider the political feasibility situation, as explored by Seguino, Sumner, van der Hoeven, Sen, and Ahmed [2013] in their survey study, I can restrict each pair $(x, A)$

[^3]to be that $x \in A$. Here, prospect set $A$ is explained as a set of allocations that can be implemented through the political system. ${ }^{4}$ Alternatively, when we consider the ideal allocation study as Almås et al. [2011], Cappelen et al. [2007], I can restrict prospect set to be singleton.

Recall that we say that a function $I: X \rightarrow \mathbb{R}$ represents an index of objective inequality if (i) $0 \leq I(x) \leq 1$ for all $x \in X$; (ii) $I(x)=0$ iff $x=c \cdot \mathbb{1}$ where $c>0$ and $\mathbb{1}$ is a unit vector in $X$; and (iii) $I(x)<I(y)$ while $x$ is a Pigou-Dalton transfer of $y .{ }^{5}$

Definition 1. A function $J: \mathbb{D} \rightarrow \mathbb{R}$ is an index of the perception of inequality if there exists an index of objective inequality $I$ such that for $(x, A) \in \mathbb{D}$,

$$
\begin{equation*}
J(x, A)=\left|I(x)-\theta \min _{y \in A} I(y)\right|, \tag{1}
\end{equation*}
$$

where $0 \leq \theta \leq 1$. In particular, we say $J$ is a Gini index of the perception of inequality if $I$ is an objective Gini coefficient defined as follows: for all $x \in X$,

$$
\begin{equation*}
I_{g}(x)=\frac{\sum_{1 \leq i<j \leq n}\left|x_{i}-x_{j}\right|}{n \mu(x)} \tag{2}
\end{equation*}
$$

According to the definition, the source of the individual perception of inequality is twofold. One is the objective inequality of the real income profile, which is measured by $I(x)$. The other is the prospect equality, which is measured by $\min _{y \in A} I(y)$. The parameter $\theta$ measures the degree to which prospect equality affects perception. When $\theta=0$, the perception coincides with the objective inequality. When $\theta=1$, the perceived equality is perfect if objective equality and prospect equality coincide.

In fact, discounted prospect equality, $\theta \min _{y \in A} I(y)$, may represent individual beliefs of fairness, as in Alesina and Angeletos [2005], and any actual allocation

[^4]deviating from it will be regarded as unfair allocation. When objective inequality is greater than prospect equality, individuals perceive inequality and believe that greater reallocation would improve equality. When objective inequality is less than prospect equality, individuals still perceive inequality but reject redistribution.

A final remark about my proposed index. It has been widely and long acknowledged that the perception of inequality is well affected by individuals' own income. However, the discrepancy between objective and perceived inequality is not related to individual income. To highlight such discrepancies, I assume that individuals only know the actual allocation and the prospect set but do not know what income they will be allocated to.

Proposition 1. The function $J$ defined in eq (1) satisfies the following properties:
(i) for all $(x, A) \in \mathbb{D}, 0 \leq J(x, A) \leq 1$;
(ii) $J(x, A)=0$ if and only if $I(x)=\theta \min _{y \in A} I(y)$;
(iii) if $x$ is a Pigou-Dalton transfer of $y$, then $J(x,\{x, y\})<J(y,\{x, y\})$.

This proposition first says that function $J$ lies between zero and one as the objective measure. The second property says that if objective and discounted prospect inequalities are evaluated equally, then the perception of equality is perfect whenever two inequality values are the same. The final property confirms that Pigou-Dalton transfer also improves the perception of equality whenever the prospect set consists of the two income profiles. The proposition shows that our proposed measure $J$ has the plausible properties that are appreciated in the literature of inequality.

Now, I illustrate an example to demonstrate that the proposed index is consistent with the observation that, in some situations, an objectively fairer allocation is perceived as less equal than an objectively less fair allocation. Consider a pair $(x, A)$, for simplicity, where $x \in A$. Then, we can rewrite eq (1) in the following way:

$$
J(x, A)=(1-\theta) I(x)+\theta\left(I(x)-\min _{y \in A} I(y)\right)
$$

In this way, we can see clearly that when the actual profile is among the prospect sets, the individual perception of inequality is a weighted sum of objective inequality and the shortfall in objective inequality from prospect equality. If objective
inequality is the same as prospect equality, then the perception of inequality is the same as that of objective inequality. However, if the objective inequality is larger than the prospect equality, then the perceived inequality is beyond the objective inequality. Therefore, an alternative with high objective equality and higher prospect equality may perceived as having more inequality than another alternative with high objective inequality and much higher prospect inequality. The next example illustrates this point numerically.

Example 1. A society consists of two individuals. There are four possible income profiles: $x=(7,3), y=(5,5), x^{\prime}=(9,1)$ and $y^{\prime}=(8,2)$. Their corresponding Gini indices are $I_{g}(x)=0.4, I_{g}(y)=0, I_{g}\left(x^{\prime}\right)=0.8$ and $I_{g}\left(y^{\prime}\right)=0.6$. Consider two alternatives $(x,\{x, y\})$ and $\left(x^{\prime},\left\{x^{\prime}, y^{\prime}\right\}\right)$. In the first situation, $x$ is the final income profile, which is more equal than $x^{\prime}$ under objective measure $I_{g}$. Additionally, the prospect equality in the first situation $\{x, y\}$, which is $\min \left\{I_{g}(x), I_{g}(y)\right\}=0$, is more equal than the prospect equality in the second situation $\left\{x^{\prime}, y^{\prime}\right\}$, which is 0.6 . However, the high prospect in the first situation counteracts the high objective equality, which may lead to low perceived equality. Formally, let $\theta=0.8$,
$J(x,\{x, y\})=I_{g}(x)-0.8 \times I_{g}(y)=0.4>0.32=I_{g}\left(x^{\prime}\right)-0.8 \times I_{g}\left(y^{\prime}\right)=J\left(x^{\prime},\left\{x^{\prime}, y^{\prime}\right\}\right)$.

Hence, the perceived inequality from $(x,\{x, y\})$ is larger than that from $\left(x^{\prime},\left\{x^{\prime}, y^{\prime}\right\}\right)$, although the objective inequality of $x$ is less than that of $x^{\prime}$.

## 3 Characterization of perception measurement

In this section, I set forth the ethical axioms of egalitarian relations and discuss how the principles shape the individual perception of inequality, which can be measured by a Gini index of perception of inequality. In particular, I am interested in the Gini index because of its simple form and wide application. I will discuss the extension of the Gini index after the statement of the results.

Axiom 1. (Weak order.) $\succsim$ is complete and transitive.
Axiom 2. (Continuity.) For all $(x, A) \in \mathbb{D}$, the sets $\{(y, B) \in \mathbb{D}:(x, A) \succsim$ $(y, B)\}$ and $\{(y, B) \in \mathbb{D}:(y, B) \succsim(x, A)\}$ are closed.

The preference relation $\succsim$ satisfying weak order and continuity appears in many divergent contexts throughout economic theory and does not need further elaboration.

The next axiom makes use of the notion of a distribution of the normalized measure that weights individuals by their ranked incomes. Let $\tilde{x}$ be the income distribution obtained from $x$ by rearranging the incomes in an increasing order, i.e., $\left\{x_{1}, \ldots, x_{n}\right\}=\left\{\tilde{x}_{1}, \ldots, \tilde{x}_{n}\right\}$ and $\tilde{x}_{1} \leq \ldots \leq \tilde{x}_{n}$.

Definition 2. If $n \geq 3$ and $x \in X$, then the function $L_{x}$, for $p \in[0,1]$ and $k=$ $0,1, \ldots, n$, defined by

$$
L_{x}(p)=\frac{1}{n \mu(x)} \sum_{i=1}^{k} \tilde{x}_{i} \quad \text { if } \frac{k}{n} \leq p<\frac{k+1}{n}
$$

is called the Lorenz measure associated with $x$, and its graph is referred to as the corresponding Lorenz curve.

For each $x \in X, L_{x}$ is increasing and satisfies $L_{x}(p) \leq p$ for all $p$ with $L_{x}(0)=$ 0 and $L_{x}(1)=1$. For every income profile $x, y \in X$, we say profile $x$ Lorenz dominates $y$ if $L_{x}(p) \geq L_{y}(p)$ for all $p$. We say income profile shows perfect equality, denoted by $x^{*}$, if it Lorenz dominates every income profile, i.e. $L_{x}(p)=\frac{k}{n}$ for all $\frac{k}{n} \leq p<\frac{k+1}{n}$. Clearly, if $x$ be such that $x_{i}=x_{j}$ for all $i, j$, then $x$ is a perfectly equal allocation. Note that an allocation $x$ Lorenz dominates every allocation if and only if $x$ is perfect equality.

Axiom 3. (Lorenz principle.) If $x$ Lorenz dominates $y$, then $(x,\{x\}) \succsim(y,\{y\})$. For perfect equality, $x^{*}$ and all $y \in X,\left(x^{*},\left\{x^{*}\right\}\right) \succsim\left(x^{*},\{y\}\right)$ and $\left(x^{*},\left\{x^{*}\right\}\right) \succsim$ ( $y,\left\{x^{*}\right\}$ ).

The Lorenz principle consists of two parts. The first part simply says that if one income profile Lorenz dominates the other, then the former profile, which is itself the prospect, is preferred to the latter, which is also itself the prospect. This statement actually includes two classic principles assumed in the inequality literature. First, if an income profile is a permutation of another profile, then they must have the same Lorenz measures. Our principle requires two permuted alternatives $(x,\{x\})$
and $(y,\{y\})$ to be indifferent, which implies symmetry. Second, if an income profile is a Pigou-Dalton transfer of another profile, ${ }^{6}$ then the Lorenz measure of the former dominates that of the latter. Therefore, our principle requires the former to be preferred to the latter, which satisfies the Pigou-Dalton principle.

The second part of the Lorenz principle says that if $x^{*}$ Lorenz dominates each income profile, then a pair of an arbitrary allocation and a perfectly equal prospect or a pair of a perfect equality and an arbitrary prospect singleton, is perceived as less equal compared to the perfectly equal alternative $\left(x^{*},\left\{x^{*}\right\}\right)$. In other words, if an individual prospects perfect equality, then actual allocation, which is also perfect equality, is naturally preferred to any other allocation. Similarly, if the actual allocation is perfectly equal, then any nonperfectly equal prospect is not preferred to the perfectly equal prospect.

To state the next axiom, I need some notation. I say that an alternative $(x,\{y\})$ is underprospect if $\left(x^{*},\{y\}\right) \succsim\left(x,\left\{x^{*}\right\}\right)$; overprospect if $\left(x,\left\{x^{*}\right\}\right) \succsim\left(x^{*},\{y\}\right)$; and ideal prospect if $\left(x,\left\{x^{*}\right\}\right) \sim\left(x^{*},\{y\}\right)$. If an alternative with singleton prospect is underprospect, then the 'distance' between prospect $\{y\}$ and perfect equality $x^{*}$ is smaller than the 'distance' between actual allocation $x$ and the perfectly equal prospect $\left\{x^{*}\right\}$. That is, the actual allocation $x$ is less equal than prospect equality $\{y\}$. Similarly, if the actual allocation $x$ is more equal than prospect equality $\{y\}$, then such alternative is overprospected. Finally, if both $x$ and $\{y\}$ are perceived to be of the same equality, then $(x,\{y\})$ is ideal prospect. Note that individuals may not perceive $x$ and $\{x\}$ as the same equality because the inequality measure of prospect is discounted, as displayed in eq (1).

Axiom 4 (Monotonicity.) For all ideal prospect alternatives $(x,\{y\})$, (i) if $(x,\{x\}) \succsim$

$$
\begin{aligned}
& \left(x^{\prime},\left\{x^{\prime}\right\}\right) \succsim\left(x^{\prime \prime},\left\{x^{\prime \prime}\right\}\right), \text { then }\left(x^{\prime},\{y\}\right) \succsim\left(x^{\prime \prime},\{y\}\right) ; \text { (ii) if }\left(y^{\prime},\left\{y^{\prime}\right\}\right) \succsim \\
& \left(y^{\prime \prime},\left\{y^{\prime \prime}\right\}\right) \succsim(y,\{y\}), \text { then }\left(x,\left\{y^{\prime \prime}\right\}\right) \succsim\left(x,\left\{y^{\prime}\right\}\right) .
\end{aligned}
$$

This axiom has two parts: one is about the actual income profile, and the other is about the prospect allocation. It says that for an ideal prospect alternative $(x,\{y\})$, if the actual income profile becomes more unequal relative to $x$, then individuals would perceive more inequality. This means that the actual allocation becomes

[^5]more unequal compared to perceived prospect $\{y\}$, and then the discrepancy between the actual and the prospect allocation becomes larger, in which case, individuals perceive more inequality. Additionally, if the prospect allocation becomes more equal relative to $\{y\}$, then individuals would perceive it as being more unequal. When the actual allocation is invariant, improving prospect equality would make the current allocation further from what individuals expect. Therefore, as a result, individuals would perceive more inequality, accompanied by an increase in such disappointment.

The next axiom makes use of two notions. The first is the notion of a nondecreasing order of incomes in $X$. For $c>0$, let

$$
X_{c}=\{x \in X: \mu(x)=c\}
$$

be the set of income profiles wherein each profile has the same average income $c$, and define

$$
\tilde{X}_{c}=\left\{x \in X_{c}: x_{1} \leq x_{2} \leq \cdots \leq x_{n}\right\}
$$

to be the set of income profiles whose income average is $c$ and income order is nondecreasing. The second notion is the mixture of alternatives. For $x, y \in \tilde{X}_{c}$ and $\alpha \in[0,1]$, we define

$$
z:=\alpha x+(1-\alpha) y
$$

by $z_{i}=\alpha x_{i}+(1-\alpha) y_{i}$ for $i=1, \ldots, n$.
Axiom 5. (Order-preserving Independence) For $c>0$ and $x, x^{\prime}, y, y^{\prime}, z, z^{\prime} \in \tilde{X}_{c}$, if $\left(x,\left\{x^{\prime}\right\}\right),\left(y,\left\{y^{\prime}\right\}\right)$ and $\left(z,\left\{z^{\prime}\right\}\right)$ are all underprospect (or all overprospect), then $\left(x,\left\{x^{\prime}\right\}\right) \succsim\left(y,\left\{y^{\prime}\right\}\right)$ implies $\left(\alpha x+(1-\alpha) z,\left\{\alpha x^{\prime}+(1-\alpha) z^{\prime}\right\}\right) \succsim$ $\left(\alpha y+(1-\alpha) z,\left\{\alpha y+(1-\alpha) z^{\prime}\right\}\right)$ for all $\alpha \in[0,1]$.

Axiom 5 corresponds to the comonotonic independence axiom of the Choquet expected utility theory under uncertainty (Schmeidler [1989]) and requires that the ranking between two underprospect (overprospect) alternatives is invariant with respect to a certain mixture of order-preserving underprospect (overprospect) alternatives. Consider three underprospect alternatives $\left(x,\left\{x^{\prime}\right\}\right),\left(y,\left\{y^{\prime}\right\}\right)$ and $\left(z,\left\{z^{\prime}\right\}\right)$
that have the same average incomes and are order-preserving. If $\left(x,\left\{x^{\prime}\right\}\right)$ is preferred to $\left(y,\left\{y^{\prime}\right\}\right)$, then this axiom states that any mixture of $\left(x,\left\{x^{\prime}\right\}\right)$ and $\left(z,\left\{z^{\prime}\right\}\right)$ is preferred to the same mixture of $\left(y,\left\{y^{\prime}\right\}\right)$ and $\left(z,\left\{z^{\prime}\right\}\right)$. This means that identical mixing on the mean constant and order-preserving underprospect alternatives do not affect the ranking of any pair of alternatives. A similar statement applies for the overprospect alternatives.

In fact, most axioms in the inequality literature are stated as a description of income transfer among individuals not as an income mixture. I use an example to illustrate the meaning of axiom 5, which can be equivalently derived through tax collection and redistribution. Since the mixture of prospect sets is an income-wise mixture, it is sufficient to discuss the mixture of two income profiles. Let $x$ and $y$ be income profiles with identical means and the same order preserves. Now suppose that these income profiles are affected by the following tax and transfer rule. First, a proportional tax with tax rate $1-\alpha$ is introduced. Second, a redistribution rule, in which the ranking of the post-transfer and pre-transfer must be invariant, is introduced. This means that the collected taxes are redistributed according to the scale $1-\alpha$ of some order-preserved income profile $z$. Note that the same tax rate imposed on two income profiles $x$ and $y$ would lead to the identical tax collection, which is $(1-\alpha) n c$. The redistribution of collected taxes is based on the same rule, which is $(1-\alpha) z$. It is understood that $(1-\alpha) z_{i}$ is the transfer received by the individual $i$. At one extreme, $z=(c / n, \ldots, c / n)$ is a completely equalized income profile, and the transfers to each individual are the same and equal to the average tax $(1-\alpha) c$. At the other extreme, $z=(0, \ldots, 0,(1-\alpha) c n)$, and all the collected tax will transfer to the richest individual in society. Therefore, the mixture of two income profiles has two consequences: (i) for the tax collecting $\alpha x$, the taxes paid by the poor cannot exceed those paid by the rich; and (ii) for the post-tax profile $\alpha x+(1-\alpha) z$, the income position of each individual is invariant. Hence, if $\left(x,\left\{x^{\prime}\right\}\right)$ is preferred to $\left(y,\left\{y^{\prime}\right\}\right)$, then Axiom 5 states that any order-preserving tax-transfer reform, as illustrated above, will not affect the ranking of the two alternatives.

The next axiom is motivated by the specific transfer method by Ben Porath and Gilboa [1994], in which the mean of every feasible income is also constant. For $x \in X$ and $1 \leq i, j \leq n$, we say $i$ precedes $j$ in $x$ if $x_{i} \leq x_{j}$ and there is no
$1 \leq k \leq n$ such that $x_{i}<x_{k}<x_{j}$.
Axiom 6. (Ben Porath-Gilboa Transfer Principle.) For $c>0$, take any $x, y, x^{\prime}, y^{\prime} \in$ $X_{c}$ and $1 \leq i, j \leq n$. If (a) $i$ precedes $j$ in $x, y, x^{\prime}, y^{\prime}$ and (b) $x_{i}=x_{i}^{\prime}+s$, $x_{j}=x_{j}^{\prime}-s$ and $y_{i}=y_{i}^{\prime}+s, y_{j}=y_{j}^{\prime}-s$ for some $s>0$ and (c) $x_{k}=x_{k}^{\prime}$ and $y_{k}=y_{k}^{\prime}$ for $k \notin\{i, j\}$ are satisfied, then

$$
(x,\{x\}) \succsim(y,\{y\}) \text { if and only if }\left(x^{\prime},\left\{x^{\prime}\right\}\right) \succsim\left(y^{\prime},\left\{y^{\prime}\right\}\right) .
$$

In this axiom, I consider a specific transfer, namely, a Ben Porath-Gilboa transfer of income, in which a transfer is made in two income profiles between a pair of individuals who have the same adjacent ranks in both profiles. This axiom requires that when comparing income profiles with self-prospects, the ranking of the posttransfer profiles is invariant to the ranking of pre-transfer profiles. An implication of the axiom is that the direction of preference is invariant when the compared incomes and associated self-prospects are changed by Ben Porath-Gilboa transfers. It is interesting to note that by successive applications of this axiom, we can extend to an arbitrary number of transfers that would possibly involve every individual.

We say prospect set $A$ dominates $B$ if for each $x \in A$ there exists $y \in B$ such that $(x,\{x\}) \succsim(y,\{y\})$. We say prospect sets $A$ and $B$ are equivalent if $A$ dominates $B$ and $B$ dominates $A$.

Axiom 7 (Equivalence.) For all $x$ and $A, B$, if $A$ and $B$ are equivalent, then

$$
(x, A) \sim(x, B)
$$

The final axiom states that if two alternatives have the same income allocation and equivalent prospect sets, then they are indifferent.

Now, we state the main result of this paper, which is a characterization of a Gini measure of the perception of inequality.

Theorem 1. An individual preference relation $\succsim$ satisfies Axioms 1-7 if and only if there exists $J$ as in eqs (1) and (2) that represents $\succsim$.

The theorem states that Axioms 1-7 provide a complete characterization for the evaluation of the perception of inequality. The perception of inequality is represented by an absolute shortfall of the discounted prospect equality from an objective
income inequality. Moreover, both inequalities are evaluated by the most common objective instrument, which is the Gini index. In fact, the objective measure of $I$ does not necessarily take the Gini form; I can easily extend it by using different forms of function $I$. For example, an alternative form, the so-called linear measure, which generalizes the Gini coefficient, is given by

$$
I(x)=\frac{\sum_{i=1} \beta_{i} \tilde{x}_{i}}{\mu(x)}
$$

for $\beta_{1}>\beta_{2}>\cdots>\beta_{n}$ for all $x \in X$. One may easily verify that by restricting Axiom 6 on set $\tilde{X}_{c}$, our set of axioms can characterize representation function $J$ as in eq (1), where $I$ has a linear measure form.

Since preferences over income profiles are most noteworthy in the field of redistribution policy, I next discuss how the social perception of inequality, being an aggregation of individual perceptions, affects tax or redistribution policy. Suppose, for simplicity, that both individuals and society take the range of policy feasibility as prospect sets. Therefore, the prospect set is uniform across all individuals and contains pretax income allocation and all possible after-tax income allocations. Society follows the utilitarian aggregation rule, and therefore, social preferences are represented by $J$ on $\{(x, A) \in \mathbb{D}: x \in A\}$ as in eq (1). Now, let $x \in X$ be the pretax income profile. Consider the choice of tax policy. A tax scheme $t$ is a function from $X$ to $X$ such that $\mu(t(x))=\mu(x)$ for all $x \in X$. Let $T$ be a set of feasible tax schemes that are politically acceptable. Therefore, a prospect set is given by

$$
A=\{t(x): t \in T\} .
$$

Suppose $y$ is the post-tax income profile generated by some tax scheme $t$. We can re-write $J$ in the following way:

$$
J(y, A)=(1-\theta) I(y)+\theta\left(I(y)-\min _{z \in A} I(z)\right)
$$

If the social planner selects a tax scheme such that $I(y)=\min _{z \in A} I(z)$, then the social perception of inequality is the same as the objective inequality of the posttax income profile. However, if $I(y)>\min _{z \in A} I(z)$, then the social perception
of inequality consists of two parts: the objective inequality of the post-tax income profile and the shortfall of prospect equality from post-tax inequality. In fact, the latter is somehow related to Arthur Okun's famous metaphor. The measure $I(y)-$ $\min _{z \in A} I(z)$ can be interpreted as how "leaky" a "bucket" a social planner is willing to accept. Based on this explanation, my notion can distinguish leaky buckets from inequality reduction, which is measured by $I$. This provides a sharp contrast to the idea that both inequality reduction and leaky buckets are represented by the form of $I$ as in Yaari [1988] and many followers.

## 4 Voting On REDISTRIBUTION

In this section, I will show, through a voting mechanism on redistribution, how prospect inequality preferences lead to a situation in which an objectively more equal society may demand more redistribution compared to an objectively less equal society. Now, I will assume that before voting, each voter (individual) knows exactly the income allocated to her. Therefore, the voters utility consists of both the self-interest part and the perception of inequality part. Formally, for $i=1, \ldots, n$, we say voter $i$ has prospect inequality preferences over $\mathbb{D}$ if her utility function $u_{i}: \mathbb{D} \rightarrow \mathbb{R}$ has the following form:

$$
\begin{equation*}
u_{i}(x, A)=x_{i}-\delta \cdot\left|I_{g}(x)-\theta \min _{y \in A} I_{g}(y)\right| . \tag{3}
\end{equation*}
$$

where scalars $0 \leq \delta, \theta \leq 1$. To simplify my analysis, I assume that voters are homogeneous on $\delta$ and $\theta$. The parameter $\delta$ captures how much the voter cares about the perception of inequality. When $\delta=0$, my model coincides with the model with self-interested voters only, as in Meltzer and Richard [1981]. The parameter $\theta$, as I discussed at length above, captures how much the prospect affects voters' perception of inequality. When $\theta=0$, my model is, in spirit, consistent with inequality aversion models, such as Fehr and Schmidt [1999].

To incorporate my model into the voting mechanism, I assume that each voter $i$ must belong to one of two income classes, either rich or poor. Further, I assume that the rich income class has $n_{r}$ voters and the poor income class has $n_{p}$ voters such that
$n_{r}+n_{p}=n$. I denote by $x=\left(x_{r}, x_{p}\right)$ the pretax income allocation, where $x_{r}>x_{p}$. There are only two possible types of prospect allocations. Low prospect voters believe that effort determines the current allocation and dislike a reduction in the inequality level. That is, the low prospect set contains only pretax income allocation $\{x\}$. In contrast, high prospect voters believe that luck determines allocation and expect a reduction in the inequality level. That is, the high prospect set contains only perfectly equal allocation $\left\{x^{*}\right\}$. Let $n_{r \ell}$ denote the number of rich voters who are of the low prospect type and $n_{p \ell}$ denote the number of poor voters who are of the low prospect type. Therefore, the number of rich voters who are of the high prospect type is $n_{r h}=n_{r}-n_{r \ell}$, and the number of poor voters who are of the high prospect type is $n_{p h}=n_{p}-n_{p \ell}$. Note that if there is no low prospect type, then all voters have objective inequality aversion preferences, which in spirit is consistent with Fehr and Schmidt [1999]. These assumptions allow us to simplify the above utility function.
(1). The pretax utility of rich voters with low prospects is

$$
u_{r \ell}(x)=x_{r}-\delta\left|I_{g}(x)-I_{g}(x)\right|=x_{r}
$$

(2). The pretax utility of rich voters with high prospects is

$$
u_{r h}(x)=x_{r}-\delta\left|I_{g}(x)-I_{g}\left(x^{*}\right)\right|=x_{r}-\delta I_{g}(x)
$$

(3). The pretax utility of poor voters with low prospects is

$$
u_{p \ell}(x)=x_{p}
$$

(4). The pretax utility of poor voters with high prospects is

$$
u_{p h}(x)=x_{p}-\delta I_{g}(x)
$$

Consider uniform redistribution policy $0<t \leq 1$. If this policy is adopted through voting, then each voter $i$ must pay a tax $t x_{i}$. The total collected tax is
$\left(n_{r} x_{r}+n_{p} x_{p}\right) t$. Therefore, each voter will receive transfer $b=\frac{\left(n_{r} x_{r}+n_{p} x_{p}\right) t}{n}$. I denote by $x(t)$ the after-tax income allocation when the tax rate is $t$. Therefore, a voter would vote 'yes' for policy $t$ if and only if her after-tax utility is higher than her pretax utility. Note that the objective inequality of after-tax income allocation is always smaller than that of pretax allocation, i.e., $I_{g}(x(t))<I_{g}(x)$.
( $1^{\prime}$ ). The after-tax utility of rich voters with low prospects is

$$
u_{r \ell}(x(t))=(1-t) x_{r}+b-\delta\left|I_{g}(x(t))-I_{g}(x)\right|
$$

Therefore, a rich voter with a low prospect will vote for tax policy $t$ if and only if

$$
\delta<\frac{b-t x_{r}}{I_{g}(x)-I_{g}(x(t))}
$$

Since $b-t x_{r}<0$ and $\delta \geq 0$, a rich voter with a low prospect will never vote for redistribution.
(2'). The after-tax utility of rich voters with high prospects is

$$
u_{r h}(x(t))=(1-t) x_{r}+b-\delta I_{g}(x(t))
$$

Therefore, a rich voter with a high prospect will vote for tax policy $t$ if and only if

$$
\delta>\frac{t x_{r}-b}{I_{g}(x)-I_{g}(x(t))}
$$

Therefore, if a rich voter with a high prospect is sufficiently sensitive to perceive inequality, then she will vote for tax policy $t$.
(3'). The after-tax utility of poor voters with low prospects is

$$
u_{p \ell}(x(t))=(1-t) x_{p}+b-\delta\left|I_{g}(x)-I_{g}(x(t))\right|
$$

Therefore, a poor voter with a low prospect will vote for tax policy $t$ if and only if

$$
\delta<\frac{b-t x_{p}}{I_{g}(x)-I_{g}(x(t))} .
$$

Contrary to the above case, if a poor voter with a low prospect is overly sensitive to the perception of inequality, then she will not vote for redistribution.
(4'). The after-tax utility of poor voters with high prospects is

$$
u_{p h}(x(t))=(1-t) x_{p}+b-\delta I_{g}(x(t)) .
$$

It is immediately clear that a poor voter with a high prospect will always vote for redistribution.

For $q \in(0,1]$, a voting mechanism is said to be a $q$-majority voting rule if the number of voters who vote for policy $t$ must be greater than $q n$ for the policy to be accepted.

Proposition 2. Consider a tax policy $t \in(0,1]$.
(i) If $n_{p}<n_{r}$ and $\delta \in\left(\frac{t x_{r}-b}{I_{g}(x)-I_{g}(x(t))}, \frac{b-t x_{p}}{I_{g}(x)-I_{g}(x(t))}\right)$, then tax policy $t$ is accepted iff $n_{r h}+n_{p}>q n$.
(ii) If $n_{p}<n_{r}$ and $\delta>\frac{b-t x_{p}}{I_{g}(x)-I_{g}(x(t))}$, then tax policy $t$ is accepted iff $n_{r h}+n_{p h}>q n$.
(iii) If $n_{p}>n_{r}$ and $\delta>\frac{t x_{r}-b}{I_{g}(x)-I_{g}(x(t))}$, then tax policy $t$ is accepted iff $n_{r h}+n_{p h}>$ $q n$.
(iv) If $n_{p}>n_{r}$ and $\delta<\frac{b-t x_{p}}{\left.I_{g}(x)-I_{g}(x)(t)\right)}$, then tax policy $t$ is accepted iff $n_{p}>q n$.

This proposition characterizes four situations concerning the acceptance of a redistribution policy. Item (iv) says that when a society has more poor than rich voters, if voters are not sensitive to the perception of inequality, then redistribution is accepted whenever the number of poor voters is greater than the $q$ majority. This result is similar to that of the Meltzer-Richard model. When $\delta$ is sufficiently small, the prospect inequality preference is close to the self-interest preference. Therefore,
under the majority voting rule where $q=1 / 2$, redistribution is accepted whenever the income of the median is less than the average income, which means when there are more poor voters than rich voters.

Item (iii) demonstrates that a society, where the poor is more than the rich, may not vote for redistribution if voters are more sensitive to the perception of inequality. When $\delta$ is high, there are opposing effects between the rich with high prospects and the poor with low prospects. If rich voters have a strong fair mind, i.e. the high prospect type, then they would support redistribution to reduce inequality. On the other hand, poor voters who have weak fair mind, i.e. the low prospect type, would vote against redistribution to prevent an increase in perceived inequality. Therefore, whether a redistribution will be accepted depends on the numerical difference between the rich with high prospects and the poor with low prospects. Hence, an objectively very unequal society, with more poor than rich people, may block redistribution if there are enough poor voters with weak fair minds.

When a society has more rich voters, Meltzer and Richard [1981] showed that self-interest voters would never support redistribution. However, items (i) and (ii) show that this is not necessarily true when voters have prospect inequality preferences. Item (ii) characterizes the redistribution condition when $\delta$ is high. As I analyze above the effect of large $\delta$, whether redistribution is accepted depends on the number of voters with high prospects. That is, if a society has more voters with high prospects and all voters are sufficiently sensitive to perceive inequality, then, regardless of the ratio of rich and poor, redistribution is always appreciated. Comparing items (ii) and (iii), we can see that a less equal society, where $n_{p}>n_{r}$, may block redistribution if there are enough voters with low prospects. In contrast, a more equal society, where $n_{r}>n_{p}$, may support redistribution if voters with high prospects dominate. Therefore, this proposition offers a possible explanation for the redistribution puzzle, as Alesina and Angeletos [2005] and many others have observed.

If $\delta$ is not too large, then a weak fair-minded poor voter may prefer redistribution to improve her material income and not be overly disappointed about an increase in perceived inequality. Therefore, when this is the case, item (i) shows that if rich voters with low prospects are not dominant, then redistribution is always accepted.

In fact, there have already been several studies about voting on redistribution where voters are assumed to be inequality averse, as in Fehr and Schmidt [1999]. These all found that fair-minded voters can well explain a demand for redistribution even when society is relatively equal, which is not compatible with the MeltzerRichard model. Unfortunately, neither the Meltzer-Richard nor inequality aversion model can explain why a relatively unequal society does not demand redistribution. However, equipped with prospect inequality preferences, item (iii) of Proposition 2 demonstrates that this can happen if there are enough poor voters with a weak fair mind.

## 5 Concluding Remarks

It has been long acknowledged that objective inequality differs from perceived inequality. In this paper, an axiomatic method on individual perception is proposed and demonstrated to characterize an index for the perception of inequality. This method suggests that the formation of perception consists of objective and prospect (in)equalities. If social preference, being an aggregation of individual preferences, is also a prospect inequality preference, then it can well explain the redistribution puzzle of why a society with better objective equality requires more redistribution than a society with worse objective equality. Alternatively, I consider voting on redistribution and demonstrate how voters with prospect inequality preferences can resolve the redistribution puzzle.

For a long time, economists did not favor the use of subjective data for axiomatic purposes. However, the empirical study cited in this paper shows that individuals answer survey questions to properly express their prospect equality. By and large, the subjective data provide the critical information needed to understand voting behavior, which objective data does not. I agree that further progress is needed to improve the way in which we now derive prospect equality. However, the imperfectness of subjective data should not be an excuse for abandoning it.

## Appendix: Proofs

## Proof of Proposition 1

Suppose that $J$ has the form as in eq (1). To see (i), take arbitrarily $(x, A) \in \mathbb{D}$. It is clear that $J \geq 0$. Suppose first, $I(x) \geq \theta \min _{y \in A} I(y)$. Since $I(x) \leq 1$ and $\min _{y \in A} I(y)>0$, we have $J(x, A)=I(x)-\theta \min _{y \in A} I(y) \leq 1$. Suppose that $I(x)<\theta \min _{y \in A} I(y)$. Then, $-\theta \min _{y \in A} I(y)<I(x)-\min _{y \in A} I(y) \leq 0$. Hence, $J(x, A) \leq \theta \min _{y \in A} I(y) \leq 1$. Clearly, (ii) holds trivially. To see (iii), assume that $x$ is a Pigou-Dalton transfer of $y$. So, $I(x)<I(y)$. It is immediate to have $J(x,\{x, y\})<J(y,\{x, y\})$.

## Proof of Theorem 1

The proof of necessity part is routine and, therefore, I omit it. I only prove the suffiency part. Our proof consists of three parts. First, I show the existence of representation. Second, I restrict on the set $\mathbb{D}_{c}$ and show that preferences on it can be represented by a Gini index of perception of inequality. Finally, I extend the representation to the whole domain $\mathbb{D}$.

Lemma 1. There exists a continuous function $J: \mathbb{D} \rightarrow \mathbb{R}$ that represents $\succsim$ on $\mathbb{D}$.
Proof. Since the preference $\succsim$ on $\mathbb{D}$ is a weak order and satisfies continuity, the Debreu Theorem implies that there must exist a continuous function $J$ that represents $\succsim$.

Definition 3. A preference relation $\succsim^{*}$ on $X_{c}$ is a $B G$-preference if the following hold:
(i) $\succsim^{*}$ is a weak order;
(ii) $\succsim^{*}$ is continuous: the sets $\left\{y: x \succsim^{*} y\right\}$ and $\left\{y: y \succsim^{*} x\right\}$ are closed.
(iii) $\succsim^{*}$ is symmetric: if $x$ is a permutation of $y$, then $x \sim^{*} y$;
(iv) $\succsim^{*}$ satisfies Pigou-Dalton Transfer Principle: for all $x, y \in \tilde{X}_{c}$ and all $i \neq n$, if $x_{j}=y_{j}$ for all $j \notin\{i, i+1\}$ and for some $s>0, x_{i}=y_{i}+s$ and $x_{i+1}=y_{i+1}-s$, then $x \succsim^{*} y$.
(v) $\succsim^{*}$ satisfies order-preserving transfer: for all $x, y, x^{\prime}, y^{\prime} \in X_{c}$, if (a) $i$ precedes $j$ in $x, y, x^{\prime}, y^{\prime}$ and (b) $x_{i}=x_{i}^{\prime}+s, x_{j}=x_{j}^{\prime}-s$ and $y_{i}=y_{i}^{\prime}+s, y_{j}=y_{j}^{\prime}-s$ for some $s>0$ and (c) $x_{k}=x_{k}^{\prime}$ and $y_{k}=y_{k}^{\prime}$ for $k \notin\{i, j\}$, then $x \succsim^{*} y$ if and only if $x^{\prime} \succsim^{*} y^{\prime}$.

Let $x^{*}=(c, c, \cdots, c)$ be the allocation that each individual has the same income $c$. Clearly, $x^{*}$ is a perfect equality allocation. We define objective inequality preference $\succsim_{I}$ on $X_{c}$ by for all $x, y \in X_{c}$,

$$
x \succsim_{I} y \Leftrightarrow\left(x,\left\{x^{*}\right\}\right) \succsim\left(y,\left\{x^{*}\right\}\right)
$$

Lemma 2. Objective inequality preference $\succsim_{I}$ is a $B G$ preference.
Proof. By definition, it is immediate to see that $\succsim_{I}$ is a weak order and satisfies continuity. Since every $x$ has the same Lorenz measure as its permutation, then Axiom 3 implies that if $y$ is a permutation of $x$, then $(x,\{x\}) \sim(y,\{y\})$. Monotonicity implies that $\left(x,\left\{x^{*}\right\}\right) \sim\left(y,\left\{x^{*}\right\}\right)$. Therefore, by definition $x \sim_{I} y$, which demonstrate that symmetry property holds. To see Pigou-Dalton transfer principle, let $x, y \in \tilde{X}_{c}$ be such that there is $i \neq n$ such that $x_{j}=y_{j}$ for all $j \notin\{i, i+1\}$ and for some $s>0, x_{i}=y_{i}+s$ and $x_{i+1}=y_{i+1}-s$. Note that for any $p \in[0,1]$ and $p \notin\left[\frac{i}{n}, \frac{i+1}{n}\right)$,

$$
L_{x}(p)=L_{y}(p)
$$

But, for $p \in\left[\frac{i}{n}, \frac{i+1}{n}\right)$,

$$
L_{x}(p)=\frac{1}{n \mu(x)} \sum_{k=1}^{i} \tilde{x}_{k}=\frac{1}{n \mu(y)}\left[\sum_{k=1}^{i-1} \tilde{y}_{k}+y_{i}+s\right]>L_{y}(p) .
$$

Since the Lorenz measure of $x$ is higher than that of $y, x$ Lorenz dominates $y$. By Axiom 3, $(x,\{x\}) \succsim(y,\{y\})$, which implies $\left(x,\left\{x^{*}\right\}\right) \sim\left(y,\left\{x^{*}\right\}\right)$ by monotonicity. Therefore, Pigou-Dalton Transfer Principle is satisfied. Finally, orderpreserving transfer follows straightforward from Axiom 4 and monotonicity.

Accordint to Ben Porath and Gilboa [1994] Theorem B, there exists a function
$u: X_{c} \rightarrow \mathbb{R}$, defined by for all $x \in X_{c}$

$$
u(x)=\alpha \sum_{1 \leq i<j \leq n}\left|x_{i}-x_{j}\right|+\beta,
$$

where $\alpha>0$ and $\beta$ is a real number such that $x \succsim_{I} y$ if and only if $u(x) \leq u(y)$.
I now define prospect inequality preference $\succsim_{P}$ on $X_{c}$ by for all $x, y \in X_{c}$,

$$
x \succsim_{P} y \Leftrightarrow\left(x^{*},\{y\}\right) \succsim\left(x^{*},\{x\}\right)
$$

Lemma 3. The prospect equality preference $\succsim_{P}$ is a $B G$-preference.
Proof. The proof that $\succsim_{P}$ is a weak order and satisfies continuity is routine. To see symmetry, note that if $x$ is a permutation of $y$, then Lorenz measure of $x$ and $y$ are the same. Therefore, Axiom 3 implies that $(x,\{x\}) \sim(y,\{y\})$. Monotonicity implies that $\left(x^{*},\{x\}\right) \sim\left(x^{*},\{y\}\right)$, which is $x \sim_{P} y$. To see Pigou-Dalton transfer principle, let $x, y \in \tilde{X}_{c}$ be such that there is $i \neq n$ such that $x_{j}=y_{j}$ for all $j \notin\{i, i+1\}$ and for some $s>0, x_{i}=y_{i}+s$ and $x_{i+1}=y_{i+1}-s$. By the calculation in the above proof, we know that the Lorenz measure of $x$ is higher than that of $y$. Hence $x$ Lorenz dominates $y$. By Axiom 3, $(x,\{x\}) \succsim(y,\{y\})$. Monotonicity further implies that $\left(x_{*},\{x\}\right) \succsim\left(x_{*},\{y\}\right)$. Hence, Pigou-Dalton Transfer Principle is satisfied. Finally Axiom 4 and monotonicity implies order-preserving transfer holds for $\succsim_{P}$.

Again, according to Ben Porath and Gilboa [1994] Theorem B, there exists a function $v: X_{c} \rightarrow \mathbb{R}$, defined by for all $x \in X_{c}$

$$
v(x)=\alpha^{\prime} \sum_{1 \leq i<j \leq n}\left|x_{i}-x_{j}\right|+\beta^{\prime},
$$

where $\alpha^{\prime}>0$ and $\beta^{\prime}$ is a real number such that $x \succsim_{P} y$ if and only if $v(x) \leq v(y)$. Finally, note that $u(x)$ and $v(x)$ are cardinally equivalent.

Let

$$
\tilde{\mathbb{D}}_{c}^{+}=\left\{(x,\{y\}) \in \mathbb{D}: x, y \in \tilde{X}_{c} \text { and }(x,\{y\}) \text { is under-prospect. }\right\},
$$

and

$$
\tilde{\mathbb{D}}_{c}^{-}=\left\{(x,\{y\}) \in \mathbb{D}: x, y \in \tilde{X}_{c} \text { and }(x,\{y\}) \text { is over-prospect. }\right\} .
$$

Lemma 4. The sets $\tilde{\mathbb{D}}_{c}^{+}$and $\tilde{\mathbb{D}}_{c}^{-}$are convex.
Proof. I first show that $\tilde{\mathbb{D}}_{c}^{+}$is convex. According to the above results, I can plot indifference curves of $(x,\{y\})$ in $\widetilde{\mathbb{D}}_{c}^{+}$on $(u, v)$ plane as in Figure (1), in which $u$ axis represents $u(x)$ and $v$ axis represents $v(y)$. Let $(x,\{y\}) \in \tilde{\mathbb{D}}_{c}^{+}$. Since it is under prospect, we know that, for perfect equality $x^{*} \in \tilde{X}_{c},\left(x^{*},\{y\}\right) \succsim\left(x,\left\{x^{*}\right\}\right)$. According to monotonicity, $\left(x^{*},\left\{x^{*}\right\}\right) \succsim\left(x^{*},\{y\}\right)$ and $\left(x,\left\{x^{*}\right\}\right) \succsim\left(x_{*},\left\{x^{*}\right\}\right)$ for perfect inequality $x_{*}=(0,0, \ldots, n c)$. Continuity implies that there exists $\alpha \in[0,1]$ such that $\left(x^{*},\{y\}\right) \sim\left(\alpha x^{*}+(1-\alpha) x_{*},\left\{x^{*}\right\}\right)$. Let $y^{*}:=\alpha x^{*}+(1-\alpha) x_{*}$. So, $\left(y^{*},\{y\}\right)$ is ideal prospect by definition and, hence, is in $\tilde{\mathbb{D}}_{c}^{+}$. Therefore, for any $y^{\prime} \in \tilde{X}_{c}$, if $\left(y^{*},\left\{x^{*}\right\}\right) \succsim\left(y^{\prime},\left\{x^{*}\right\}\right)$, or equivalently $u\left(y^{*}\right) \leq u\left(y^{\prime}\right)$, then $\left(y^{\prime},\{y\}\right) \in$ $\tilde{\mathbb{D}}_{c}^{+}$.

Now, I want to show that if $\left(y^{*},\{y\}\right)$ is ideal prospect, then, for all $\alpha \in[0,1]$, $\left(\alpha x^{*}+(1-\alpha) y^{*},\left\{\alpha x^{*}+(1-\alpha) y\right\}\right)$ is also ideal prospect. Suppose not, there are two cases to consider:
Case 1: There is ideal prospect $\left(z^{*},\{z\}\right)$, which lies on the right of ideal prospect line connecting $\left(x^{*},\left\{x^{*}\right\}\right)$ and $\left(y^{*},\{y\}\right)$. Since both $\left(y^{*},\{y\}\right)$ and $\left(z^{*},\{z\}\right)$ are idea prospect, we know $\left(y^{*},\left\{x^{*}\right\}\right) \sim\left(x^{*},\{y\}\right)$ and $\left(z^{*},\left\{x^{*}\right\}\right) \sim\left(x^{*},\{z\}\right)$. However, by monotonicity, we must have $\left(y^{*},\left\{x^{*}\right\}\right) \succ\left(z^{*},\left\{x^{*}\right\}\right) \sim\left(x^{*},\{z\}\right) \succ\left(x^{*},\{y\}\right)$, which is a contradiction.

Case 2: There is ideal prospect $\left(w^{*},\{w\}\right)$, which lies on the left of ideal prospect line connecting $\left(x^{*},\left\{x^{*}\right\}\right)$ and $\left(y^{*},\{y\}\right)$. Similar argument implied to the Case 1 will lead to a contradiction. Therefore, all the ideal prospect must lie on the same line connecting to $\left(x^{*},\left\{x^{*}\right\}\right)$.

Now, we know that all the under prospect lie on the right of ideal prospect line. To see $\tilde{\mathbb{D}}_{c}^{+}$is convex, take two under-prospect $\left(y^{\prime},\{y\}\right)$ and $\left(z^{\prime},\{z\}\right)$ as in the figure. Take arbitrary $\alpha \in[0,1]$, comonotonic additivity of $u$ and $v$ imply that

$$
\begin{aligned}
u\left(\alpha y^{\prime}+(1-\alpha) z^{\prime}\right) & =\alpha u\left(y^{\prime}\right)+(1-\alpha) u\left(z^{\prime}\right) \\
v(\alpha y+(1-\alpha) z) & =\alpha v(y)+(1-\alpha) v(z)
\end{aligned}
$$

Hence, point $\left(\alpha y^{\prime}+(1-\alpha) z^{\prime},\{\alpha y+(1-\alpha) z\}\right)$ must lie between the line connecting $\left(y^{\prime},\{y\}\right)$ and $\left(z^{\prime},\{z\}\right)$, which means it also belongs to set $\tilde{\mathbb{D}}_{c}^{+}$.

Finally, similar arguments as above imply that $\tilde{\mathbb{D}}_{c}^{-}$is also convex.
Lemma 5. The function $J$ restricted on $\tilde{\mathbb{D}}_{c}^{+}$has the following form: for all $(x,\{y\}) \in$ $\tilde{\mathbb{D}}_{c}^{+}$,

$$
J(x,\{y\})=I(x)-\theta_{1} I(y),
$$

where $I$ is a Gini index and $\theta_{1} \geq 0$.
Proof. First, restrict $\succsim$ on $\widetilde{\mathbb{D}}_{c}^{+}$. Let $\left(x,\left\{x^{\prime}\right\}\right)$ and $\left(y,\left\{y^{\prime}\right\}\right)$ be in $\widetilde{\mathbb{D}}_{c}^{+}$such that $\left(x,\left\{x^{\prime}\right\}\right) \sim\left(y,\left\{y^{\prime}\right\}\right)$. I claim that in the figure the straight line that connects point $\left(x,\left\{x^{\prime}\right\}\right)$ and point $\left(y,\left\{y^{\prime}\right\}\right)$ is an indifference curve. According to Axiom 5, we know that for all $\alpha \in[0,1]$,

$$
\left(\alpha x+(1-\alpha) y,\left\{\alpha x^{\prime}+(1-\alpha) y^{\prime}\right\}\right) \sim\left(y,\left\{y^{\prime}\right\}\right) .
$$

Comonotonic additivity of $u$ implies that

$$
u(\alpha x+(1-\alpha) y)=\alpha u(x)+(1-\alpha) u(y) .
$$

Similarly, comonotonic additivity of $v$ implies that

$$
v\left(\alpha x^{\prime}+(1-\alpha) y^{\prime}\right)=\alpha v\left(x^{\prime}\right)+(1-\alpha) v\left(y^{\prime}\right)
$$

Therefore, it is clear to see that the point $\left(\alpha x+(1-\alpha) y,\left\{\alpha x^{\prime}+(1-\alpha) y^{\prime}\right\}\right)$ in the $(u, v)$ planes lies between point $\left(x,\left\{x^{\prime}\right\}\right)$ and $\left(y,\left\{y^{\prime}\right\}\right)$. Hence, all the indifference curves in the figure are straight lines.

I then claim that any indifference curves in the figure parallel to each other. To see this, take any $\left(z,\left\{z^{\prime}\right\}\right) \in \widetilde{\mathbb{D}}_{c}^{-}$which is not indifferent to $\left(x,\left\{x^{\prime}\right\}\right)$. According to Axiom 5, we know that for all $\alpha \in[0,1]$,

$$
\left(\alpha x+(1-\alpha) z,\left\{\alpha x^{\prime}+(1-\alpha) z^{\prime}\right\}\right) \sim\left(\alpha y+(1-\alpha) z,\left\{\alpha y^{\prime}+(1-\alpha) z^{\prime}\right\}\right)
$$

Comonotonic additivity of $u$ implies that

$$
\begin{aligned}
& u(\alpha x+(1-\alpha) z=\alpha u(x)+(1-\alpha) u(z) \\
& u(\alpha y+(1-\alpha) z=\alpha u(y)+(1-\alpha) u(z)
\end{aligned}
$$

Also, the comonotonic additivity of $v$ implies that

$$
\begin{aligned}
& v\left(\alpha x^{\prime}+(1-\alpha) z^{\prime}\right)=\alpha v\left(x^{\prime}\right)+(1-\alpha) v\left(z^{\prime}\right) \\
& v\left(\alpha y^{\prime}+(1-\alpha) z^{\prime}\right)=\alpha v\left(y^{\prime}\right)+(1-\alpha) v\left(z^{\prime}\right)
\end{aligned}
$$

In Figure (1), it is clear that point $\left(\alpha x+(1-\alpha) z, \alpha x^{\prime}+(1-\alpha) z^{\prime}\right)$ and point $\left(\alpha y+(1-\alpha) z, \alpha y^{\prime}+(1-\alpha) z^{\prime}\right)$ in $(u, v)$ plane are between the line connecting $\left(x,\left\{x^{\prime}\right\}\right),\left(z,\left\{z^{\prime}\right\}\right)$ and the line connecting $\left(y,\left\{y^{\prime}\right\}\right),\left(z,\left\{z^{\prime}\right\}\right)$, respectively. Furthermore, we know that both $\left(\alpha x+(1-\alpha) z, \alpha x^{\prime}+(1-\alpha) z^{\prime}\right)$ and $(\alpha y+(1-$ $\left.\alpha) z, \alpha y^{\prime}+(1-\alpha) z^{\prime}\right)$ are indifferent. Therefore, the line connecting both points is an indifference curve. By elementary geometry, we know immediately that both indifference curves are parallel to each other.

Since indifference curves in $\widetilde{\mathbb{D}}_{c}^{+}$are straight and parallel, and $\widetilde{\mathbb{D}}_{c}^{+}$is convex, the representation function $J$ on $\widetilde{\mathbb{D}}_{c}^{+}$should have the following form: for all $(x,\{y\}) \in$ $\widetilde{\mathbb{D}}_{c}^{+}$,

$$
J(x,\{y\})=a \cdot u(x)+b \cdot v(y),
$$

where $a$ and $b$ are real numbers. Recall the form of function $u$ and $v$ on $X_{c}$. Therefore, we can normalize the two functions such that

$$
\begin{aligned}
J(x,\{y\}) & =a \cdot\left[\alpha \sum_{1 \leq i<j \leq n}\left|x_{i}-x_{j}\right|+\beta\right]+b \cdot\left[\alpha^{\prime} \sum_{1 \leq i<j \leq n}\left|y_{i}-y_{j}\right|+\beta^{\prime}\right] \\
& =a \cdot \alpha\left[\sum_{1 \leq i<j \leq n}\left|x_{i}-x_{j}\right|+\frac{b \cdot \alpha^{\prime}}{\alpha} \sum_{1 \leq i<j \leq n}\left|y_{i}-y_{j}\right|\right]+\left(\beta+\beta^{\prime}\right)
\end{aligned}
$$

Define function $I$ on $X_{c}$ by

$$
I(x)=\frac{1}{n^{2} c} \sum_{1 \leq i<j \leq n}\left|x_{i}-x_{j}\right|
$$

Let $\beta+\beta^{\prime}=0, \alpha=1 / n^{2} c$ and $\alpha^{\prime}=\alpha^{2}$. Then, the normalized $J$ can be written in the following form: for all $(x,\{y\}) \in \widetilde{\mathbb{D}}_{c}^{+}$,

$$
J(x,\{y\})=a \cdot I(x)+b \cdot I(y) .
$$

Now, I want to show that $a>0$ and $b \leq 0$. Note that $\left(x^{*},\left\{x^{*}\right\}\right),\left(x_{*},\left\{x^{*}\right\}\right) \in \widetilde{\mathbb{D}}_{c}^{+}$. Therefore, $\left(x^{*},\left\{x^{*}\right\}\right) \succsim\left(x_{*},\left\{x^{*}\right\}\right)$ implies that

$$
a \cdot I\left(x^{*}\right)+b \cdot I\left(x^{*}\right) \leq a \cdot I\left(x_{*}\right)+b \cdot I\left(x^{*}\right) .
$$

Since $I\left(x^{*}\right)=0$ and $I\left(x_{*}\right)>0$, we must have $a>0$. For ideal prospect $\left(y^{*},\{y\}\right) \in \tilde{\mathbb{D}}_{c}^{+}$, let $\left(y^{\prime},\left\{y^{\prime}\right\}\right),\left(y^{\prime \prime},\left\{y^{\prime \prime}\right\}\right)$ be such that $\left(y^{\prime},\left\{y^{\prime}\right\}\right) \succsim\left(y^{\prime \prime},\left\{y^{\prime \prime}\right\}\right) \succsim$ $(y,\{y\})$. Clearly, both $\left(y^{*},\left\{y^{\prime}\right\}\right)$ and $\left(y^{*},\left\{y^{\prime \prime}\right\}\right)$ are under-prospect. Monotonicity implies that $\left(y^{*},\left\{y^{\prime \prime}\right\}\right) \succsim\left(y^{*},\left\{y^{\prime}\right\}\right)$, which is

$$
a \cdot I\left(y^{*}\right)+b \cdot I\left(y^{\prime \prime}\right)<a \cdot I\left(y^{*}\right)+b \cdot I\left(y^{\prime}\right)
$$

Hence, $b \leq 0$. Hence, by normalization,

$$
J(x,\{y\})=I(x)-\theta_{1} I(y),
$$

where $\theta_{1}$ is a real number, represents $\succsim$ restricted on $\tilde{\mathbb{D}}_{c}^{+}$.
Lemma 6. The function $J$ restricted on $\tilde{\mathbb{D}}_{c}^{-}$has the following form: for all $(x,\{y\}) \in$ $\tilde{\mathbb{D}}_{c}^{-}$,

$$
J(x,\{y\})=-I(x)+\theta_{2} I(y),
$$

where $I$ is a Gini index and $\theta_{2} \geq 0$.
Proof. First, we repeat the proof of the above lemma and can obtain that for all $(x,\{y\}) \in \widetilde{\mathbb{D}}_{c}^{-}$,

$$
J(x,\{y\})=a \cdot I(x)+b \cdot I(y)
$$

where $a$ and $b$ are real numbers. Now, we want to show that $a<0$. Note that
$\left(x^{*},\left\{x^{*}\right\}\right),\left(x^{*},\left\{x_{*}\right\}\right) \in \widetilde{\mathbb{D}}_{c}^{-}$. Therefore, $\left(x^{*},\left\{x^{*}\right\}\right) \succsim\left(x^{*},\left\{x_{*}\right\}\right)$ implies that

$$
a \cdot I\left(x^{*}\right)+b \cdot I\left(x^{*}\right) \leq a \cdot I\left(x^{*}\right)+b \cdot I\left(x_{*}\right) .
$$

Since $I\left(x^{*}\right)=0$ and $I\left(x_{*}\right)>0$, we must have $b \geq 0$. Since ideal prospect $\left(y^{*},\{y\}\right) \in \tilde{\mathbb{D}}_{c}^{-}$, let $\left(y^{\prime},\left\{y^{\prime}\right\}\right),\left(y^{\prime \prime},\left\{y^{\prime \prime}\right\}\right)$ be such that $\left(y^{\prime},\left\{y^{\prime}\right\}\right) \succsim\left(y^{\prime \prime},\left\{y^{\prime \prime}\right\}\right) \succsim$ $\left(y^{*},\left\{y^{*}\right\}\right)$. Since $\left(y^{\prime},\{y\}\right),\left(y^{\prime \prime},\{y\}\right)$ are both over prospect, monotonicity implies that $\left(y^{\prime \prime},\{y\}\right) \succsim\left(y^{\prime},\{y\}\right)$, which is

$$
a \cdot I\left(y^{\prime \prime}\right)+b \cdot I(y) \leq a \cdot I\left(y^{\prime}\right)+b \cdot I(y)
$$

Since $I\left(y^{\prime \prime}\right)>I\left(y^{\prime}\right)$, we must have $a<0$. Therefore, by normalization,

$$
J(x,\{y\})=-I(x)+\theta_{2} I(y),
$$

where $\theta_{2}$ is a real number, represents $\succsim$ restricted on $\tilde{\mathbb{D}}_{c}^{-}$.
Lemma 7. $0 \leq \theta_{1}=\theta_{2} \leq 1$.
Proof. We first want to show that $\theta_{1}=\theta_{2}$. Notice that the intersection of $\tilde{\mathbb{D}}_{c}^{+}$and $\tilde{\mathbb{D}}_{c}^{-}$is the set of ideal prospect. According to the representation restricted on $\tilde{\mathbb{D}}_{c}^{+}$, for $z \in \tilde{X}_{c}$, if $I(z)$ is small enough, there exists $w \in \tilde{\mathbb{D}}_{c}^{+}$such that

$$
I(w)-\theta_{1} \cdot I(z)=0
$$

So, the pair $(w,\{z\})$ is ideal prospect and is indifferent to $\left(x^{*},\left\{x^{*}\right\}\right)$, otherwise there exists $w^{\prime}$ such that $I\left(w^{\prime}\right)<I(w)$ and $I\left(w^{\prime}\right)-\theta_{1} \cdot I(z)<0$. Since ideal prospect also belongs to $\tilde{\mathbb{D}}_{c}^{-}$, we also have $-I(w)+\theta_{2} \cdot I(z)=0$. Therefore, $\theta_{1}=\theta_{2}$.

Note that if there is $x \in \tilde{X}_{c}$ such that $(x,\{x\}) \in \tilde{\mathbb{D}}_{c}^{+}$, then for all $y \in \tilde{X}_{c}$, $(y,\{y\}) \in \tilde{\mathbb{D}}_{c}^{+}$. Otherwise, all $(y,\{y\}) \in \tilde{\mathbb{D}}_{c}^{-}$. Now, suppose that $(x,\{x\}) \in \tilde{\mathbb{D}}_{c}^{+}$. Take $y \in \tilde{X}_{c}$ be such that $(x,\{x\}) \succsim(y,\{y\})$. According to the representation restricted on $\tilde{\mathbb{D}}_{c}^{+}$,

$$
I(x)-\theta \cdot I(x) \leq I(y)-\theta \cdot I(y)
$$



Figure 1: Indifference curve.

Positivity of $I$ implies that $\theta \leq 1$. Similar argument implies to the case where $(x,\{x\}) \in \tilde{\mathbb{D}}_{c}^{-}$.

Lemma 8. The representation function $J$ restricted on $\tilde{\mathbb{D}}_{c}^{+} \cup \tilde{\mathbb{D}}_{c}^{-}$coincides with a Gini index of perception of inequality as in eq (1) and (2).

Proof. Take $\left(x,\left\{x^{\prime}\right\}\right) \in \tilde{\mathbb{D}}_{c}^{+}$and $\left(y,\left\{y^{\prime}\right\}\right) \in \tilde{\mathbb{D}}_{c}^{-}$. I want to show that $\left(x,\left\{x^{\prime}\right\}\right) \succsim$ ( $y,\left\{y^{\prime}\right\}$ ) if and only if

$$
I(x)-\theta \cdot I\left(x^{\prime}\right) \geq-I(y)+\theta \cdot I\left(y^{\prime}\right)
$$

Since $\left(x,\left\{x^{\prime}\right\}\right) \in \tilde{\mathbb{D}}_{c}^{+}$, there exists $z \in \tilde{X}_{c}$ such that, for perfect equality $x^{*}$, $\left(z,\left\{x^{*}\right\}\right) \in \tilde{\mathbb{D}}_{c}^{+}$and

$$
I(x)-\theta \cdot I\left(x^{\prime}\right)=I(z)-\theta \cdot I\left(x^{*}\right)
$$

Note that $0 \leq I(x)-\theta \cdot I\left(x^{\prime}\right) \leq 1$. Therefore, the existence of $z$ is guaranteed. Hence, $\left(x,\left\{x^{\prime}\right\}\right) \sim\left(z,\left\{x^{*}\right\}\right)$. Similarly, there exists $w \in \tilde{X}_{c}$ such that $\left(x^{*},\{w\}\right) \in$ $\tilde{\mathbb{D}}_{c}^{-}$and

$$
-I(y)+\theta \cdot I\left(y^{\prime}\right)=-I\left(x^{*}\right)+\theta \cdot I(w) .
$$

I claim that $\left(z,\left\{x^{*}\right\}\right) \succsim\left(x^{*},\{w\}\right)$, which is $I(z) \geq \theta \cdot I(w)$. Suppose not, assume $\left(x^{*},\{w\}\right) \succ\left(z,\left\{x^{*}\right\}\right)$. Since $\left(y,\left\{y^{\prime}\right\}\right) \sim\left(x^{*},\{w\}\right)$ and $\left(z,\left\{x^{*}\right\}\right) \sim$ $\left(x,\left\{x^{\prime}\right\}\right)$, this leads to a contradiction of assumption. Hence, $I(z) \geq \theta \cdot I(w)$, which is equivalent to $I(x)-\theta \cdot I\left(x^{\prime}\right) \geq-I(y)+\theta \cdot I\left(y^{\prime}\right)$. The case for $\left(y,\left\{y^{\prime}\right\}\right) \succsim\left(x,\left\{x^{\prime}\right\}\right)$ is very similar and we omit the proof.

Now, we know for any $c>0$, there exists a Gini index of perception of inequality that represents $\succsim$ on $\tilde{\mathbb{D}}_{c}$. Furthermore, due to the Axiom 3, there exists a $\theta$ such that $0 \leq \theta \leq 1$ such that $\theta=-\theta_{c}$ for all $c>0$. We consider $\succsim$ on the set $\tilde{\mathbb{D}}_{c}=\bigcup_{c>02} \tilde{\mathbb{D}}_{c}$.

Lemma 9. The function $J$ defined on eq (1) and (2) represents $\succsim$ on $\tilde{\mathbb{D}}$.
Proof. Now consider $\succsim$ on $\tilde{\mathbb{D}}=\bigcup_{c>0} \tilde{\mathbb{D}}_{c}$. We show that $J$ restricted on $\tilde{\mathbb{D}}$ represents $\succsim$ on $\succsim \mathbb{D}$. Let $c, e>0$. Take any $\left(x,\left\{x^{\prime}\right\}\right) \in \tilde{\mathbb{D}}_{c}$ and $\left(y,\left\{y^{\prime}\right\}\right) \in \tilde{\mathbb{D}}_{e}$ such that $\left(x,\left\{x^{\prime}\right\}\right) \succsim\left(y,\left\{y^{\prime}\right\}\right)$. Let $\alpha=c / e$. Then $\mu(\alpha y)=\mu\left(\alpha y^{\prime}\right)=c$. So $\left(\alpha y,\left\{\alpha y^{\prime}\right\}\right) \in$ $\mathbb{D}_{c}$. It is immediate to see that $y$ and $\alpha y$ have the same Lorenz measure and so are $y^{\prime}$ and $\alpha y^{\prime}$. By Axiom 3, $\left(y,\left\{y^{\prime}\right\}\right) \sim\left(\alpha y,\left\{\alpha y^{\prime}\right\}\right)$. Transitivity implies that $\left(x,\left\{x^{\prime}\right\}\right) \succsim\left(\alpha y,\left\{\alpha y^{\prime}\right\}\right)$, which is

$$
I(x)-\theta I\left(x^{\prime}\right) \leq I(\alpha y)-\theta I\left(\alpha y^{\prime}\right)
$$

Note that

$$
\begin{aligned}
I(y) & =\frac{\sum_{1 \leq i<j \leq n}\left|y_{i}-y_{j}\right|}{n^{2} e} \\
& =\frac{\sum_{1 \leq i<j \leq n}\left|\alpha y_{i}-\alpha y_{j}\right|}{\alpha n^{2} c / \alpha} \\
& =I(\alpha y) .
\end{aligned}
$$

Then, it implies that

$$
\left(x,\left\{x^{\prime}\right\}\right) \succsim\left(y,\left\{y^{\prime}\right\}\right) \Leftrightarrow I(x)-\theta I\left(x^{\prime}\right) \leq I(y)-\theta I\left(y^{\prime}\right)
$$

For all $A \in \mathscr{A}$, define $I(A)=\min _{x \in A} I(x)$.
Lemma 10. For all $(x, A) \in \mathbb{D}$ and $y \in \tilde{X}_{c}$ with $c=n \mu(x)$, if $I(A)=I(y)$, then $(x, A) \sim(x,\{y\})$.
Proof. From the above discussion, we know that for all $z \in A$, there exists $y \in \tilde{X}_{c}$ such that $I(z)=I(y)$. Now, let $y \in \tilde{X}_{c}$ be such that $I(A)=I(y)$, then $A$ and $\{y\}$ are equivalent. Therefore, by Axiom 7, we have immediately that $(x, A) \sim$ ( $x,\{y\}$ ).

From the above analysis, for each $(x, A)$, there always exists $y \in \tilde{X}_{c}$ where $c=n \mu(x)$ such that $I(A)=I(y)$. Therefore, I have shown the preference representation on $\mathbb{D}$.

## Proof of Proposition 2

First, note that the rich with low prospect will never vote for any tax policy $0<t \leq$ 1 and the poor with high prospect will always vote for any tax policy. When there is more rich, i.e. $n_{r}>n_{p}$, we have

$$
\begin{aligned}
\left(t x_{r}-b\right)-\left(b-t x_{p}\right) & =t\left(x_{r}-x_{p}\right)-2 b \\
& =t\left(x_{r}+x_{p}\right)-2 \times \frac{\left(n_{r} x_{r}+n_{p} x_{p}\right) t}{n} \\
& =\left(n x_{r}+n x_{p}-2 n_{r} x_{r}-2 n_{p} x_{p}\right) \frac{t}{n} \\
& =\left(n_{p}-n_{r}\right)\left(x_{r}-x_{p}\right) \frac{t}{n} \\
& <0
\end{aligned}
$$

Therefore, if $n_{r}>n_{p}$, then

$$
0<t x_{r}-b<b-t x_{p} .
$$

(i). If $\delta \in\left(\frac{t x_{r}-b}{I_{g}(x)-I_{g}(x(t))}, \frac{b-t x_{p}}{I_{g}(x)-I_{g}(x(t))}\right)$, then both rich with high prospect and poor with low prospect will vote for policy $t$. Therefore, the number of vote 'Yes' is $n_{r h}+n_{p}$ and $t$ is accepted iff $n_{r h}+n_{p}>q n$.
(ii).If $\left.\delta>\frac{b-t x_{p}}{I_{g}(x)-I_{g}(x(t))}\right)$, then rich with high prospect will vote for $t$, but not the poor with low prospect.

Note that if $n_{p}>n_{r}$, then $0<b-t x_{p}<t x_{r}-b$. (iii) and (iv) follows straight similarly as I show above.

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[^1]:    ${ }^{1}$ For theoretical studies, see Atkinson [1970], Kolm [1969], Sen [1997] and an excellent survey by Cowell [2000].

[^2]:    ${ }^{2}$ One way to establish the prospect level of equality is the inequality of opportunity approach, as in Roemer and Trannoy [2016].

[^3]:    ${ }^{3}$ We adopt the assumption that a society consists of a finite number of individuals. In contrast, Yaari [1988] and Aaberge [2001] deal with a continuum society.

[^4]:    ${ }^{4}$ If we put the constraint $x \in A$ in our framework, then our representation form (1) boils down to the form: $J(x, A)=I(x)-\theta \min _{y \in A} I(y)$. See the previous version of this paper Qu [2021] for the characterization result. However, under this situation, objective equality $I(x)$ is always less equal to the prospect equality $\min _{y \in A} I(y)$. Therefore, it rules out the situation that individual prospect equality is less equal to the objective equality, which, in my opinion, is a limitation.
    ${ }^{5}$ If $x, y \in X$, we say $x$ is a Pigou-Dalton transfer of $y$ if for some $i, j \in\{1, \ldots, n\}$ we have $x_{k}=y_{k}$ for $k \notin\{i, j\}$ and $x_{i}+x_{j}=y_{i}+y_{j}$ and $\left|x_{i}-x_{j}\right|<\left|y_{i}-y_{j}\right|$.

[^5]:    ${ }^{6}$ See footnote 5 for the formal definition of Pigou-Dalton transfer.

